r = separation between molecules
T = thermodynamic temperature
T^* = reduced thermodynamic temperature, $T^* = [T/(\epsilon/k)]$
V = velocity of sound
V_0 = reduction constant for velocity of sound, $V_0 = (N_0\, \epsilon/M)^{1/2}$
V^* = reduced velocity of sound, $V^* = [V/V_0]$
V_σ = velocity of sound under saturated vapor conditions
V_σ^* = reduced velocity of sound under saturated vapor conditions

Greek Letters

ϵ, σ = intermolecular potential parameters
ϕ = intermolecular potential
ρ = density

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Stability of a Laminar Viscous Jet — The Influence of the Initial Disturbance Level

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The instability and subsequent breakup of liquid jets projected into a gaseous atmosphere have been studied by many authors since the turn of the century. In particular, Fenn and Middleman (1969) have reported some significant recent experiments in which the ambient pressure for the jet was varied. Their results show some shortcomings of the stability analysis as normally applied. It has been traditional to assume that the initial disturbance level is constant with respect to exit velocity and with respect to disturbance wave number. The search has been for some new mode of instability or some extra parameter that will give an amplification rate that better fits the data. It is the purpose of this paper to examine critically the correlation of data and theory in this light. The outcome is the suggestion that the resolution of the difficulties is to correlate data by considering the initial disturbance level as variable.

THEORETICAL BACKGROUND

Jet stability theory assumes an axisymmetric, exponentially growing disturbance which ruptures the jet when its amplitude grows equal to the jet radius. As a result, the time-to-breakup T (which can also be interpreted as distance from the exit by knowing the jet velocity) is given by the relationship

$$T = \left(\frac{1}{\alpha}\right) \ln \left(\frac{D}{2\delta}\right) \tag{1}$$

Grant and Middleman (1966) give an outline of the theory and a complete summary of progress in its application to this problem.

The problem reduces to that of finding a mathematical description for the forces causing breakup, and from this calculating the maximum amplification rate as a function of the physical parameters. In a few simple cases an analytic form can be found for α. In more complex cases a computer solution must be obtained for particular conditions. It is noteworthy that the analytic solution of Weber (1931), which does not include the effects of the ambient fluid, but which does include the jet viscosity, has been very successful in correlating data with low exit velocity (where the ambient effects are small). This success has spurred a search for a more complicated solution that would correlate a wider range of data. In particular, an explanation is sought for the observed fact that there always seems to be some critical point above which the breakup length stops increasing with exit velocity and starts to decrease.

DISCUSSION OF EXPERIMENTAL RESULTS

Fenn and Middleman (1969) report a series of experiments in which the ambient pressure is reduced in steps. At each step, jet breakup is measured as a function of exit velocity. They find that at low ambient pressure the breakup is independent of pressure. As the ambient pressure is increased, its influence on the breakup could be detected. Theoretically, the influence of the ambient fluid should be described by the ambient Weber number $We_a = D\rho_a V^2/\sigma$. From the experiments, it is found that if $We_a < 5.3$ there is no effect of the ambient fluid.

When the ambient pressure is low, the condition $We_a < 5.3$ can be satisfied through the complete range of exit velocity of the experiments. In these cases the breakup curves do indeed coincide independently of the pressure. The problem is, however, that this invariant curve has a peak breakup distance with a fall-off at higher exit velocities. If this curve is independent of the ambient conditions, Weber's theory should apply; but then there doesn't seem to be any factor to which to attribute the peak. Weber's theory predicts a continual increase in breakup distance with exit velocity with no peak at all. At this point we cannot completely exclude the possibility that the data can be explained on the basis of some theory which includes a new factor other than the ambient gas density. If it exists, this theory remains to be demonstrated. In fact, as is suggested below this theory would have heavy demands placed upon it.

There is one further perplexing observation concerning experimental tests. Phinney and Humphries (1970) made a comparison between jets formed by orifice plates and by long cylindrical nozzles. It was found that at low speeds the maximum breakup length was independent of how the jet was formed. In both cases the results for the same fluid correlated with Weber's theory using the jet diameter and exit velocity. As the jet velocity increases, the orifice continues to follow the theory; however in some cases the jet from cylindrical nozzles do not. One expects that within a short distance of the exit (a few diameters at most), the velocity distribution across the jet becomes uniform and from that point on no difference between the orifice and the pipe nozzle should exist, except possibly in the initial disturbance level. Since the orifice and the pipe agree at low speeds, the conclusion must be that the initial disturbance is the same in both cases. This leaves the difference unaccounted for.

RESOLUTION OF THE DIFFICULTIES

Since Weber's theory has shown such success for most cases to which it should apply, let us assume that the amplification rate as given is correct. Instead of looking for some new theory to give a different value of α (which seems hopeless for the experiments quoted), let us explore the possibility that the above mentioned failures of the theory could be due instead to an initial disturbance level δ/D which is not constant at all speeds. The question was considered in a limited way by Grant and Middleman (1966) and Meister and Scheele (1969).

If the disturbances at the jet exit are not independent of velocity, we would expect them to depend upon the internal flow, which implies the nozzle Reynolds number for given shape. The nozzle Reynolds number Re is defined as $Re = \rho_j DV/\mu$. Since surface tension acts during the short formation time while the exit velocity profile is relaxing to its flat shape, the nondimensional parameter Z which characterizes the fluid may have a secondary influence. The Ohnesorge number Z is defined by the relation $Z = \mu/\sqrt{\rho_j D\sigma}$.

To test the above hypothesis, the data of Fenn and Middleman (1969), Grant and Middleman (1966), and Phinney and Humphries (1970) were used. The criterion of Fenn and Middleman (1969) $We_a < 5.3$ was used to sort out only those data points for which the influence of the ambient air was negligible. Use is made of Weber's theory for the relation.

$$\left(\frac{L}{D}\right)\frac{1}{\sqrt{We_j}\,(1+3Z)} = \ln\left(\frac{D}{2\delta}\right) \quad (2)$$

where L is the breakup length ($L = VT$), and We_j is the Weber number $We_j = \rho_j DV^2/\sigma$. For each data point, the initial distribution level ($2\delta/D$) from Equation (2) is computed, as well as the Reynolds number Re. The resulting data are plotted in Figures 1 through 3. As is seen at low exit velocity, we get the constant value for $2\delta/D$ that we expect (from the success of Weber's theory). At the high velocity end of the curve, the disturbance level breaks away at some point and increases ($D/2\delta$ decreases) with Re. The breakaway values of Re, $\hat{Re}$, vary somewhat from curve to curve, but they are all within the range of laminar pipe flow $Re < 2300$. Above $\hat{Re}$ a major portion of the data can be represented by $\ln(2\delta/D) \sim Re^2$ (the scatter in the data relates to the difficulty of obtaining accurate data at high exit velocity (see Grant and Middleman for comments).

To be precise, define the critical Reynolds number $\hat{Re}$

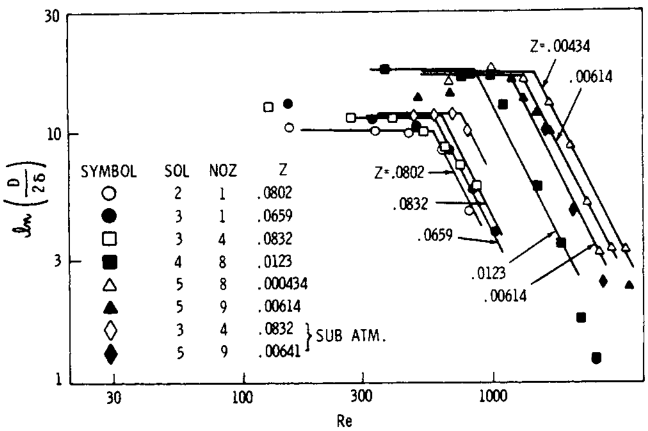

Fig. 1. Initial disturbance versus Reynolds number—Grant's data.

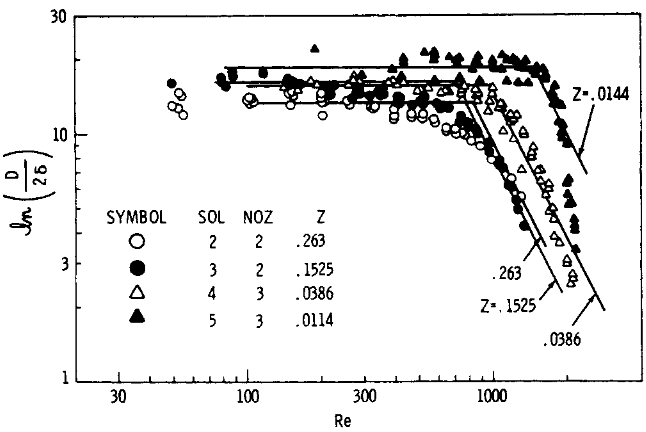

Fig. 2. Initial disturbance versus Reynolds number—Fenn's data.

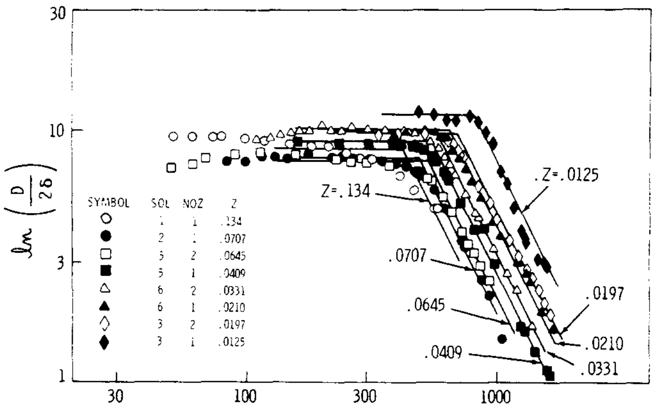

Fig. 3. Initial disturbance versus Reynolds number—NOL data.

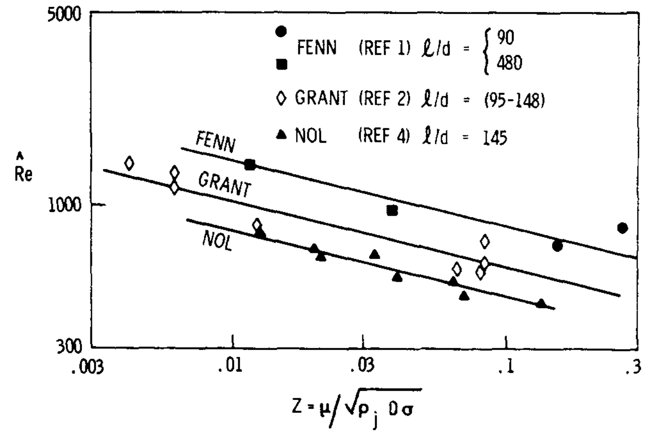

Fig. 4. Variation of critical Reynolds number with Ohnesorge number.

for any particular nozzle fluid combination by the intersection of the low speed δ/D = constant plateau with the best fitting $\ln(2\delta/D) \sim Re^2$ line. This definition allows for the fact that the low speed plateaus, although similar, are not exactly the same for all cases. If $\hat{Re}$ is plotted against Z for all three sources of data used here Figure 4 results. There are systematic differences between the different sets of experiments, but the shift in $\hat{Re}$ with Z has the same trend for the three different sources.

It should be noted that in Z the only variable that is really changed is jet viscosity. For this reason Figure 4 cannot be taken as proof that Z is necessarily the proper parameter. The fundamental relationship between Z (or whatever the proper variable might be) and $\hat{Re}$ is not understood at this point and requires particular study. As mentioned earlier, the relationship might be connected with a short interaction region near the jet exit. There is a second possibility. The relationship could be due to the fact that when the exit disturbance level increases, it need not be a uniform increase across the complete wave number range. The theoretical amplification rate for disturbances depends upon both wave number and Z. A change in Z would reflect itself as a change in amplification rate as a function of wave number which in turn would react differently with the disturbance spectrum.

CONCLUSIONS

Capillary breakup can explain the maximum in the length-velocity curve if changes in initial disturbance level are considered. This is consistent with the statement from Fenn and Middleman (1969), ". . . . jets with Weber numbers (We_a) less than 5.3 are destroyed by axisymmetric waves leading to drop formation. This is so, even beyond the maximum in the breakup curve, contrary to the notion that the maximum is associated with a change in breakup mechanism for axisymmetric-to-transverse waves." This phenomenon was also noted by Grant and Middleman (1966).

Since the peak in the length velocity curve caused by both the initial disturbance increase and the ambient atmospheric effects occur at high exit velocity, it is easy to confuse these effects. A combination of these two effects should correlate the data with $We_a > 5.3$ (the data that was not used here).

Because of a lack of data, it is not possible to investigate the characteristics of jets from an orifice in the same detail as for a pipe. It is clear from preceding discussions, however, that it is difficult to explain agreement between corresponding orifices and pipes at low speed and their divergence at high speed on any other basis than a different shift in the initial disturbance level with exit velocity.

By focusing on a different phenomenon (rather than amplification rate only), it is possible to initiate other areas of investigation. For example, since the critical Reynolds number $\hat{Re}$ is of the same order of magnitude (but lower) as that for transition-to-turbulence in the pipe, there is probably a connection between them. It is assumed basically that the same disturbances that ultimately produce turbulence in the pipe are those that also produce the increasing disturbance level at the exit for high jet velocity.

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NOTATION

D	=	jet diameter
L	=	breakup length of jet
Re	=	Reynolds number $\rho_j DV/\mu$
$\hat{Re}$	=	critical Reynolds number at which the disturbance level starts to increase
T	=	time for jet to break up
V	=	jet exit velocity
We_a	=	Weber number based on ambient density $D\rho_a V^2/\sigma$
We_j	=	Weber number based on jet density $D\rho_j V^2/\sigma$
Z	=	Ohnesorge number $\mu/\sqrt{\rho_j D\sigma}$
α	=	amplification rate
δ	=	initial disturbance in jet radius
μ	=	jet viscosity
ρ_a	=	ambient density
ρ_j	=	density of jet fluid
σ	=	surface tension

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